D) Coulomb's law

1) Point Charge

When the sizes of charged bodies are much smaller than the distance separating them, the sizes of the bodies may be ignored. These charged bodies are then called point charges. All the charge of the body can be assumed to be concentrated at one point. Thus point charge does not have length ,area or volume.

2) Coulomb's law

Coulomb,s law states that; **"The force of interaction between two point charges is directly proportional to the product of the magnitudes of the charges and inversely proportional to the square of the distance between them."**

If q_1 and q_2 are two point charges separated by a distance r in vacuum, then the force between the charges will be

$$F \propto \frac{q_1 q_2}{r^2}$$

By introducing a constant of proportionality k, we can write this as an equality,

or $$F = \frac{k q_1 q_2}{r^2} \quad \longrightarrow (1)$$

1. Both the charges experience forces. These forces are equal in magnitude and opposite in direction. The law covers both these forces.
2. The force on each particle is always directed along the line joining the two particles.
3. The forces are attractive in the case of unlike charges, and repulsive in the case of like charges.
4. The law is strictly valid only for point charges.

The proportionality constant k in eq.(1) for vacuum/free space is written as $\frac{1}{4\pi\varepsilon_0}$, where

ε_0 is called permitivity of vacuum or free space.

Its value is $\boldsymbol{\varepsilon_0 = 8.8542 \times 10^{-12}\ C^2 N^{-1} m^{-2}}$

The value of the constant k for vacuum in SI units is,

$$k = \frac{1}{4\pi\varepsilon_0} = 8.98755 \times 10^9\ Nm^2/C^2$$

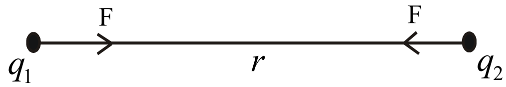
ie $$\mathbf{k \cong 9 \times 10^9\ Nm^2/C^2}$$

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \quad \xrightarrow{\text{(in vacuum)}} \quad (2)$$

3) Nature of electrostatic force

Like Newton's law of gravitation, Coulomb's law is an inverse square law. The force is inversely proportional to the square of the distance between the charges.

$$F \propto \frac{1}{r^2}$$



Many of the concepts developed for gravitation like those of field, potential, potential energy etc. are equally applicable to the electric force. In spite of these similarities, gravitation and electric interaction are fundamentally different. They are two distinct classes of phenomena. Gravitational interaction depends on mass and is always attractive. Electrical interaction depends on electric charges and can be either attractive or repulsive. Another important difference is in the relative magnitudes of the electric and gravitational forces. The electric force is enormously bigger in magnitude than the gravitational force. For example, the electric force between two electrons is 10^{42} times bigger than the gravitational force.

4) Unit of charge from Coulomb's law

The SI unit of charge is the Coulomb. We can use Coulomb's law to obtain a definition for Coulomb.

When $r = 1m$, & $q_1 = q_2 = \pm 1C$, the magnitude of force is, $F = 9 \times 10^9$ N

One coulomb is the amount of charge which when placed in vacuum at a distance of one meter from an equal and similar charge would repel it with a force of 9×10^9 Newton

Since one Coulomb is a big unit of charge, in practice we use much smaller units like milliCoulomb(mC); microCoulomb(μC) and nanoCoulomb(nC).

One milliCoulomb = 10^{-3} Coulomb
One microCoulomb = 10^{-6} Coulomb
One nanoCoulomb = 10^{-9} Coulomb

Example 1: Calculate the Coulomb force between two α-particles separated by a distance of 6.4×10^{-14} m in vacuum.

Solution: An alpha particle is a He-nucleus containing two protons and two neutrons.

$q_1 = q_2 = +2e = 2 \times 1.6 \times 10^{-19} C = 3.2 \times 10^{-19} C$, $r = 6.4 \times 10^{-14} m$

$$F = \frac{kq_1q_2}{r^2} = \frac{9 \times 10^9 \times \left(3.2 \times 10^{-19}\right)^2}{\left(6.4 \times 10^{-14}\right)^2} = \mathbf{0.225\,N}$$

Example 2: The average distance between the electron and central proton in the hydrogen atom is approximately 0.5 $\overset{\circ}{A}$. What is the magnitude of the average electrostatic force of attraction between them?

Solution: $q_1 = +1.6 \times 10^{-19}\ C$, $q_2 = -1.6 \times 10^{-19}\ C$, $r = 0.5\overset{\circ}{A} = 0.5 \times 10^{-10} m$

$$F = \frac{kq_1q_2}{r^2} = \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{\left(0.5 \times 10^{-10}\right)^2} = \mathbf{9.216 \times 10^{-8}\ N}$$

Example 3. Find the ratio of electrostatic force to that of the gravitational force between;
(i) two protons, (ii) between two electrons

Solution:(i) When two protons are kept at a distance r between them,

The electrostatic force $F_e = \frac{ke^2}{r^2}$

The gravitational force $F_g = \frac{Gm_p^2}{r^2}$

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm_p^2}$$

$$= \frac{9\times10^9 \times \left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times(1.67\times10^{-27})^2}$$

$$= 1.238\times10^{36}$$

(ii)When two electrons are kept at a distance r between them,

The electrostatic force $F_e = \frac{ke^2}{r^2}$

The gravitational force $F_g = \frac{Gm_e^2}{r^2}$

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm_e^2}$$

$$= \frac{9\times10^9 \times \left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times(9.1\times10^{-31})^2} = 4.17\times10^{42}$$

5) Coulombs Law for a medium other than air / vacuum.

The Coulomb force between two charges in vacuum is given by

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \quad \longrightarrow (1)$$

If there is matter in the space between the charges, the net force acting on each is altered because charges are induced in the molecules of the intervening medium.

If a medium other than air is inserted between the charges, then Coulomb's law is written as

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} \quad \xrightarrow{\text{(in medium)}} (2)$$

Where $\varepsilon = K\varepsilon_0$ is the absolute permittivity of the medium. K is called the **dielectric constant** of the medium. It is also called **relative permittivity** of the medium and denoted by ε_r.

$$\text{Relative Permittivity} = \frac{\text{Permittivity of a medium}}{\text{Permittivity of free space}}$$

$$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Dividing eq (2) by eq (3)

$$\frac{F_0}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r = K$$

$$F_m = \frac{F_0}{K}$$

The force between two charges in a medium is always less than the force between the same two charges kept at the same distance in vacuum, by a factor equal to the dielectric constant of the medium. Higher the value of the dielectric constant, the less will be the force between charges.

Note: ***The value of K for air is only slightly greater than 1. Therefore air may be treated just like vacuum.***

Question: Suppose two charges are kept at distance r in a medium with dielectric constant K. What should be their separation in air/vacuum so that they experience the same force?

Solution: $$F_m = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$$

Let r' be the separation in air to have the same force

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(r')^2}$$

$$F_m = F_0$$

$$\frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(r')^2}$$

$$\mathbf{r' = r\sqrt{K}}$$

ie, for a distance r in a medium with dielectric constant K, the equivalent distance in vacuum is $r\sqrt{K}$.

6) Coulomb's Law in Vector Form

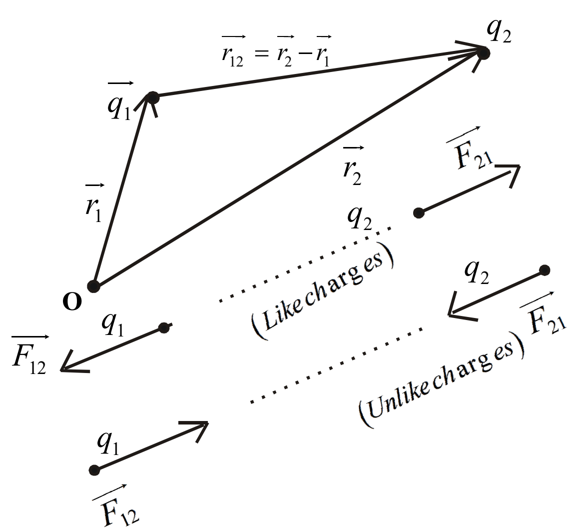
Let us now express coulomb's law in a more detailed mathematical form, using vectors. Since force is a vector quantity, we need to obtain both the magnitude and the direction of the force between two point charges. Let the two point charges q_1 and q_2 be at points with position vectors $\vec{r}_1$ and $\vec{r}_2$, w.r. to some origin O.

Vector from q_1 to $q_2 = \vec{r}_2 - \vec{r}_1 = \vec{r}_{12}$

Vector from q_2 to $q_1 = \vec{r}_1 - \vec{r}_2 = \vec{r}_{21} = -\vec{r}_{12}$

Distance between q_1 and $q_2 = r = |\vec{r}_{12}| = |\vec{r}_{21}|$

Let us denote the forces acting on the two charges in the following way;



$\vec{F}_{21}$ = force on q_2 due to q_1

$$= k\frac{q_1 q_2}{r^2}\frac{\vec{r}_{12}}{r}$$

$\vec{F}_{12}$ = force on q_1 due to q_2

$$= k\frac{q_1q_2}{r^2}\,\frac{\overrightarrow{r_{21}}}{r}$$

These equations are valid for any sign of q_1 and q_2. If both q_1 and q_2 are of the same sign(either both positive or both negative) then the forces are repulsive. If q_1 and q_2 are of opposite signs then the forces are attractive. Substituting the values of q_1 and q_2 with their signs, we get both magnitude and direction of the force from the above equations.

Further, we find the relation between $\overrightarrow{F_{12}}$ and $\overrightarrow{F_{21}}$,

$$\overrightarrow{F_{12}} = k\frac{q_1q_2}{r^2}\,\frac{\overrightarrow{r_{21}}}{r} = -k\frac{q_1q_2}{r^2}\,\frac{\overrightarrow{r_{12}}}{r} = -\overrightarrow{F_{21}} \qquad \left(\because \overrightarrow{r_{21}} = -\overrightarrow{r_{12}}\right)$$

$$\boxed{\mathbf{\overrightarrow{F_{12}} = -F_{21}}}$$

Coulomb's law is in agreement with Newton's third law of motion : The electric forces exerted by two electric charges on each other are equal in magnitude and opposite in direction. Using the notation for unit vectors, we may rewrite the expressions for the two forces using unit vectors as ;

$$\overrightarrow{F_{21}} = k\frac{q_1q_2}{r^2}\,\widehat{r_{12}} \longrightarrow (5)$$

$$\overrightarrow{F_{12}} = k\frac{q_1q_2}{r^2}\,\widehat{r_{21}} \longrightarrow (6)$$

Example 1: What is the force of repulsion between two point charges of 1C each, kept 1m apart in vaccum? What will be the force if the same charges are kept in a medium of relative permittivity $\varepsilon_r = 9$, the distance between the charges being the same?

Solution:

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \quad \text{in vaccum}$$

$$= 9\times10^9 \times \frac{1\times1}{1^2} = \mathbf{9\times10^9\ N}$$

In medium,

$$F_m = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$$

$$\frac{F_0}{\varepsilon_r} = \frac{9\times10^9}{9} N = \mathbf{10^9\ N}$$

Example 2: Two point charges of 5mC each are kept at a distance of 50 cm in water. If the force of repulsion between them is 1.125×10^{-2} N, find the dielectric constant of water.

Solution: $F = 1.125\times10^{-2}\ N \quad q_1 = q_2 = 5\times10^{-6}\ C \qquad r = 50\,cm = 0.5\,m$

$$F = \frac{kq_1q_2}{K\,r^2}$$

$$K = \frac{kq_1q_2}{F\,r^2} = \frac{9\times10^9\times\left(5\times10^{-6}\right)^2}{1.125\times10^{-2}\times\left(0.5\right)^2} = \mathbf{80}$$

Example 3: A ball A of mass 9x10^{-5} kg carries a charge of 5 μ C. What must be the magnitude and sign of the charge on a ball B held 2 cm directly above the ball A, such that the ball A remains stationary?

Solution: $m_A = 9\times10^{-5}\,kg$ $\quad q_1 = 5\times10^{-6}\ C$ $\quad AB = r = 0.02\,m$

For the ball A to be stationary

$$F_{AB} = m_A g$$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = m_A g$$

$$\frac{9\times10^9\times5\times10^{-6}\times q_2}{(0.02)^2} = 9\times10^{-5}\times9.8$$

$$q_2 = \frac{10^{-5}\times9.8}{10^7\times1.25} = \mathbf{7.84\times10^{-12}\ C}$$

q_2 must be -ve to produce a force on A, directed upwards.

Example 4: Two charged copper spheres A and B have their centers separated by a distance 50 cm in air. The charge on each sphere is 6.5×10^{-7} C. The spheres A and B have identical sizes but their radii are negligible compared to the distance between them. Suppose a third uncharged sphere of the same size is brough in contact with the first, then brought in contact with the second and finally removed to a far away place from both, what is the force of repulsion between A and B?

Solution: Original charge on A and B, $q_1 = q_2 = 6.5\times10^{-7}\,C$, when two spheres of the same size are brought in contact, they will share the charge equally. So when the third sphere is brought in contact with the first,

$$q_1' = \frac{q_1+q_3}{2} = \frac{6.5\times10^{-7}+0}{2} = 3.25\times10^{-7}\ C$$

Also $\quad q_3' = 3.25\times10^{-7}\,C$

When the third sphere is now brought in contact with the second,

$$q_2' = \frac{q_2+q_3'}{2} = \frac{6.5\times10^{-7}+3.25\times10^{-7}}{2} = 4.875\times10^{-7}\,C$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1' q_2'}{r^2} = \frac{9\times10^9\times3.25\times10^{-7}\times4.875\times10^{-7}}{(0.5)^2}$$

$$= \mathbf{5.704\times10^{-3}\ N}$$
